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When did Cantor live? What nationality was he?

Georg Ferdinand Ludwig Philipp Cantor was born on 3 March 1845 in St. Petersburg, Russia, where he lived during his first eleven years. In 1856, Cantor moved to Germany, where he remained for the rest of his life. He died on 6 January 1918 in Halle, Germany.

Did his first paper mark him as a genius?

Cantor published his first major paper in 1867 on the solution to an indeterminate equation in the form $ax^2 + by^2 + cz^2 = 0$, where a, b, and c are integers. While the treatise was thorough and careful, it did not reveal the level of genius that Cantor would later demonstrate with his studies of infinite sets.

What studies led him to attack the problem of "actual infinities"?

In 1860, Cantor's father sent him to the Höhere Gewerbeschule to study engineering. Cantor, however, was more interested in pure mathematics, and in 1862, he convinced his father to allow his transfer to the Polytechnic of Zurich for that reason. After his father's death in 1863, Cantor moved to the University of Berlin, where he met several famous mathematicians of the day, including Kronecker.

Cantor sought to dispel confusion between the "potential infinite"—when a variable grows bigger and bigger without bound—and the "actual infinite"—a fixed number representing a constant quantity.

Which mathematician led the opposition to his "actual infinities"?

Kronecker led opposition to Cantor's reasoning, which he considered insufficiently cautious. According to Kronecker, truth could only be proven by a "construction"—that is, by means of a finite number of definite and certain steps—and that existence could only be demonstrated with such a construction; Cantor, in contrast, relied heavily on nonconstructive proofs and merely assumed the existence of his infinite sets.

What factor in Cantor's earlier years contributed to his occasional breakdowns?

Cantor had always been obedient before his father's wishes and had thence never developed the self-confidence necessary to withstand criticism. As a result, when Kronecker bitterly assailed Cantor's work, Cantor was devastated. Though current medical knowledge has revealed that Cantor's depression and eventual death in a mental institution resulted from more than just attacks by fellow mathematicians, Kronecker's opposition no doubt contributed to those breakdowns.

Which mathematicians found paradoxes that threatened the theory of transfinite numbers? What does the term *antinomy* mean?

Italian mathematician Burali-Forti found a contradiction in the logic behind Cantor's infinite sets.

In 1908, Bertrand Russell pointed out the following paradox: *Consider the set of all sets that are not members of themselves. Is this set a member of itself?* Both the answers "yes" and "no" lead to a contradiction. Russell attempted to rectify his paradox with the "vicious circle principle"—that anything involving all of a collection cannot be a member of the collection.